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Evolution of the $\pi g_{9/2}\nu h^2_{11/2}$ multiplet in the neutron-rich $_{49}$In isotopes: Evidence for the gradual filling of the neutron sub-shell

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Abstract. High-spin states from the three-quasiparticle (3qp) configuration $\pi g_{9/2}\nu h^2_{11/2}$ in the neutron-rich $_{49}$In isotopes have been studied in the framework of shell model calculation. The two-body residual interactions needed for such calculations have been extracted from the level schemes of neighbouring nuclei. In particular, the evolution of the proton-neutron $\pi g_{9/2}\nu h_{11/2}$ interaction has been analyzed as a function of the neutron number. When the gradual filling of the $\nu h_{11/2}$ sub-shell is taken into account, the results of the 3qp configuration calculations are found to be very close to the experimental positive-parity yrast states located above 2 MeV excitation energy in $^{113-121}$In. Moreover, several long-lived isomeric states with positive parity are predicted in ^{123,125,127}In.

PACS. 21.60.Cs Shell model – 27.60.+j $90 \leq A \leq 149$

1 Introduction

The low-lying level spectra of the nuclei differing from doubly closed shells by two nucleons (two particles, two holes, or one particle and one hole) can be easily arranged into different pure configurations. So they have been used for a long time in order to obtain information about effective nucleon-nucleon interactions, with the aim of either deriving the components of the residual forces (see, for instance, [1–3]) or directly computing the levels of other nuclei belonging to the same valence spaces, *e.g.* the early application to the $1f_{7/2}$ nuclei [4]. This is no longer the case of the other nuclei as the increase of the number of valence nucleons leads to strong mixings between states with the same parity and spin, lying close to each other.

On the contrary, high-spin shell model states may be rather pure even though they lie several MeV above the ground state, meaning that the residual-interaction energies could be easily extracted from the corresponding excited levels and some conclusions about the residual interactions could be drawn from their study. It is however difficult to follow the same configuration along a large range of isotopes, as a lot of experimental results are missing. So only a few cases could be yet studied. For instance, neutron-proton multiplets in the odd-odd $^{106-116}$In nuclei have been studied some years ago [5]. The negative-parity bands from the $\pi g_{9/2}\nu h_{11/2}$ configuration have been analyzed in the framework of proton-neutron multiplet coupled to a quadrupole vibrational core, and the role played by the proton-neutron interaction has been discussed.

We present here the analysis of the evolution of the high-spin positive-parity states, located above 2 MeV excitation energy in neutron-rich odd-A In nuclei, as a function of the neutron number, in the framework of shell model calculations for three particles lying in two different orbits. Thanks to the high efficiency of the EUROBALL III and IV arrays, we have recently studied the gamma-ray transitions emitted by the high-spin states of several neutron-rich In nuclei produced as fission fragments in three reactions induced by heavy ions [6]. Most of the excited states observed in $^{113-121}$In have been interpreted in terms of a proton $g_{9/2}$ hole coupled to the Sn core excitation. In particular, states with spin values from $21/2^+$ to $29/2^+$ which are linked by dipole transitions having energy ranging from 100 to 500 keV, can be interpreted in terms of the three-quasiparticle (3qp) $\pi g_{9/2}\nu h^2_{11/2}$ configuration [6]. When the number of neutrons is increasing, the excitation energies of these states decrease, following

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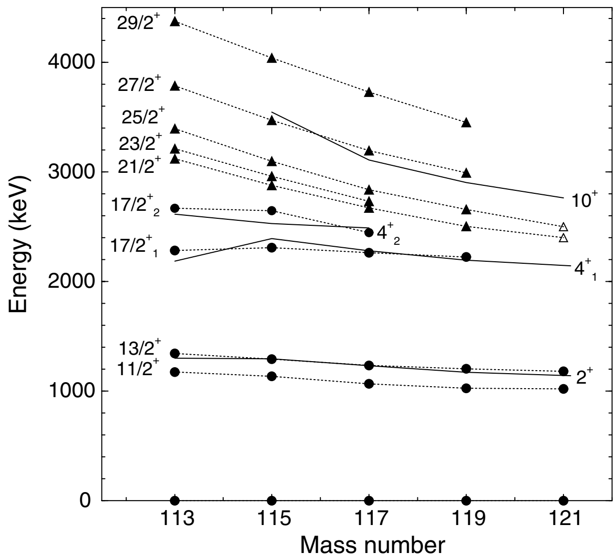

Fig. 1. Evolution of the yrast positive-parity states in odd-mass In isotopes (circles and triangles linked by dotted lines), and of the 2^+, 4^+ and 10^+ states of the corresponding Sn cores (solid lines). The energies of the $21/2^+$ and $25/2^+$ states of ^{121}In (drawn with open triangles) are not precisely known ($\Delta E \sim 50$ keV) because of two unobserved transitions [6].

the excitation energy of the 10^+ state of the Sn isotopes which is due to the breaking of one $\nu h_{11/2}^2$ pair (see fig. 1). Moreover, this configuration can be considered as almost pure, since only the $\nu h_{11/2}^2$ pair breaking can lead to spin values greater than $21/2^+$ within this range of excitation energy.

In this paper we will show how the gradual filling of the $\nu h_{11/2}$ sub-shell explains two experimental findings in the structure based on the 3qp $\pi g_{9/2} \nu h_{11/2}^2$ configuration, i) the decreasing of the dipole transition energies when A increases from 113 to 121, and ii) the disappearance of the $23/2^+$ state in ^{119,121}In. Moreover, calculations of the states based on the same configuration in the heavier masses (not known experimentally) have been performed. The existence of several long-lived isomeric states with positive parity can be expected in ^{123,125,127}In around 2.5 MeV excitation energy.

2 Shell model calculations

2.1 General features of the multi-particle states

Let us consider a configuration made of n_1 nucleons of type 1 in the j_1 sub-shell (its antisymmetrized state having a spin J_1) and n_2 nucleons of type 2 in the j_2 sub-shell (its antisymmetrized state having a spin J_2). The matrix element of the Hamiltonian for the state n_1-n_2 with the total spin J is the sum of three terms [7]:

- the individual energy of the nucleons in the j_1 sub-shell and the two-body interactions between the n_1 nucleons for the spin J_1;
- the individual energy of the nucleons in the j_2 sub-shell and the two-body interactions between the n_2 nucleons for the spin J_2;
- interactions between the n_1 nucleons in the j_1 sub-shell and the n_2 nucleons in the j_2 sub-shell, which can be written in terms of the two-body interaction between one nucleon j_1 and one nucleon j_2.

These residual two-body interactions split the degeneracy in J for the $(J_1 J_2)J$ multiplet of the states based on the n_1-n_2 configuration.

As a particular case, the relative energies of the states of the 3qp configuration, $\pi g_{9/2} \nu h_{11/2}^2$, will depend on (see eq. (31.15) of ref. [7]):

- the residual interactions in the $(\nu h_{11/2})^2$ configuration, $V_n(J_n)$ ($J_n = 0$ to 10, even values);
- the residual interactions in the $\pi g_{9/2} \nu h_{11/2}$ configuration, $V_{pn}(J_{pn})$ ($J_{pn} = 1$ to 10).

2.2 Neutron-neutron interaction: the $(\nu h_{11/2})^2$ matrix elements

The values of the 6 matrix elements $V_n(J_n)$ are obtained from the level scheme of $^{130}Sn_{80}$ as its yrast positive-parity states come from the $(\nu h_{11/2})^{-2}$ configuration. They are given in table 1.

Table 1. Energies (in keV) of the states from the $(\nu h_{11/2})^{-2}$ multiplet measured in $^{130}Sn_{80}$ [8].

J^π	0^+	2^+	4^+	6^+	8^+	10^+
E (keV)	0	1121	1996	2257	2338	2435

2.3 Proton-neutron interaction: the $\pi g_{9/2} \nu h_{11/2}$ matrix elements

The values of the 10 matrix elements V_{pn} can be obtained from the level scheme of the odd-odd In isotopes. The $\pi g_{9/2} \nu h_{11/2}$ configuration has been identified in $^{104-116}$In, its excitation energy decreases when the neutron number is increasing as the neutron Fermi level is going up into the $N = 4$ shell towards the $\nu h_{11/2}$ sub-shell. All the known experimental energies [8] are given in table 2. One can notice that states with spin 1 or 2 have never been identified in any of these odd-odd isotopes and states with spin $I \leq 7$ are unknown in the lighter isotopes.

Whereas the multiplet remains almost unchanged from ^{104}In to ^{112}In (the energies of the 9^- and 10^- states of ^{104,106,108}In are very close to the ones of ^{110,112}In, the eight energies measured in ^{110,112}In are very similar) the band spacings become more compressed from $^{112}In_{63}$ to $^{116}In_{67}$. This behaviour has to be precisely analyzed in order to get numerical values of the energies of the multiplet for the unknown $N > 67$ isotopes, which are needed to compute the 3qp states in the heaviest odd-A In nuclei.

Table 2. Energies (in keV) of the states from the $\pi g_{9/2}\nu h_{11/2}$ multiplet measured in $^{104-116}_{49}\mathrm{In}_{55-67}$ (the energy of the lowest state has been chosen as a reference) [8].

I^π	3^-	4^-	5^-	6^-	7^-	8^-	9^-	10^-
^{104}In						0	187	711
^{106}In						0	209	729
^{108}In						0	213	742
^{110}In	682	405	206	56	0	8	218	762
^{112}In	673	394	209	63	11	0	187	775
^{114}In	530	334	194	72	34	0		
^{116}In	439	265	169	83	61	0		

2.3.1 Previous study

A systematic study of the 2qp $\pi g_{9/2}\nu h_{11/2}$ multiplet in $^{106-116}$In had been done [5] in the framework of neutron-quasiparticle proton-hole quadrupole core coupling model. For the lightest masses having an occupation factor of the $\nu h_{11/2}$ sub-shell, $v^2 \sim 0$, the hole-particle configuration ($\pi g^{-1}_{9/2}\nu h^{+1}_{11/2}$) gives the typical spectrum observed in $^{104-116}$In (see table 2), i) the spin of the lowest member of the multiplet corresponds to the perpendicular coupling of the angular momenta of the proton and of the neutron, that is $\sim 7^-$, ii) the energy $E(I)$ is proportional to $I(I+1)$, the curvature being positive. When approaching the $N = 82$ closed shell ($v^2 \sim 1$), the configuration consists of two holes ($\pi g^{-1}_{9/2}\nu h^{-1}_{11/2}$). The energy $E(I)$ is still proportional to $I(I+1)$ but with a negative curvature, therefore the 10^- state is expected at the bottom of the multiplet. Such a state has been measured in $^{130}\mathrm{In}_{81}$ [8], it is a long-lived isomeric level decaying by β-emission towards high-spin states of ^{130}Sn.

One can notice that the expected change in curvature would lead to a clear compression of the band during the gradual filling of the $\nu h_{11/2}$ sub-shell. Indeed the diagonal two-body matrix elements for a proton-hole neutron-quasiparticle configuration can be written [5] as a sum of two terms, one involving the residual interaction between two particles (or identically, two holes), weighted by the occupation factor (v^2), and the other, the residual interaction between one particle and one hole, weighted by $u^2 = 1 - v^2$.

2.3.2 Present results

Following ref. [5], we have performed such a calculation for the $\pi g_{9/2}\nu h_{11/2}$ configuration. The hole-proton particle-neutron interaction is directly extracted from the experimental level scheme of ^{112}In as its neutron Fermi level is located below the $\nu h_{11/2}$ sub-shell (meaning that there is no neutron pairs in this sub-shell, $v^2 = 0$). The hole-proton hole-neutron interaction has been computed using the Pandya transformation [9], since the multiplet is not known in $^{130}\mathrm{In}_{81}$. The results are presented in fig. 2. They show the continuous evolution from the hole-particle case

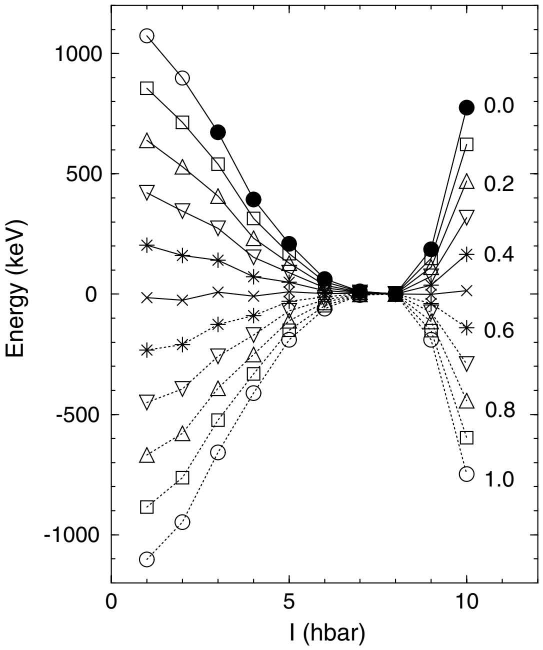

Fig. 2. Calculated energies of the states of the $\pi g_{9/2}\nu h_{11/2}$ multiplet as a function of angular momentum I, for different values of the occupation probability of the $\nu h_{11/2}$ sub-shell, $v^2 = 0.0$ to 1.0 (see text). The energies associated to $v^2 = 0$ are the experimental values measured in ^{112}In (filled circles), the $E(1^-)$ and $E(2^-)$ values (empty circles) have been extrapolated using a $I(I+1)$ law and the $E(I)$ values for $3 \le I \le 6$.

($v^2 = 0$) to the hole-hole case ($v^2 = 1$). One can notice that the states are almost degenerated in energy for $v^2 = 0.5$ and for $v^2 > 0.5$, the 10^- state is located at lower energy than the 9^-, 8^- states, leading to a long-lived isomeric state.

The experimental energies of the states measured in $^{112-116}$In [8] are drawn in the central part of fig. 3. The calculated energies extracted from fig. 2 for five values of the occupation probability of the $\nu h_{11/2}$ sub-shell ($v^2 = 0.0$ to 0.4 by step of 0.1), which are reported on the left side of fig. 3, demonstrate that the evolution of the multiplet energies in the odd-odd In isotopes having $N > 63$ is mainly governed by the gradual filling of the $\nu h_{11/2}$ sub-shell, as already noticed in ref. [5]. Therefore, all the energies drawn in fig. 2 will be used to compute the energies of the 3qp states in the odd-A In isotopes, in order to study their behaviour up to the complete filling of the $\nu h_{11/2}$ sub-shell (see sect. 3.2).

It is worth noting that the evolution of the experimental energies shows an irregularity at spin 7 in ^{114,116}In (see the central part of fig. 3). This cannot be reproduced by a sole change in the value of the $\nu h_{11/2}$ occupation probability as this effect is not seen in ^{112}In. Whereas the quadrupole components of the core-particle interaction and of the neutron-proton interaction give a parabola as a function of $I(I+1)$ for the energy splitting of the multiplet [10,5], the next-order component (for instance, a $Q_4(\pi) \cdot Q_4(\nu)$ interaction) produces some perturbation. In the particular case of the $\pi g^{-1}_{9/2}\nu h^{+1}_{11/2}$ multiplet, the

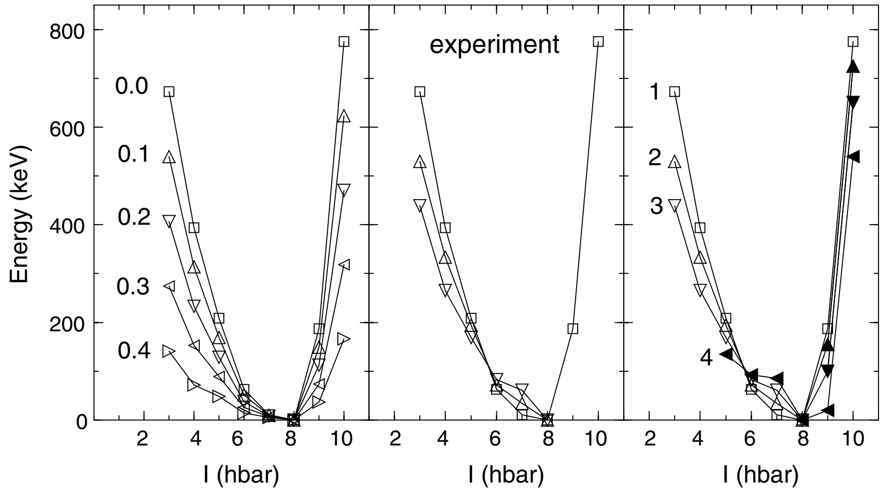

Fig. 3. Left: excitation energies of the states of the multiplet $\pi g_{9/2}\nu h_{11/2}$ for five values of the occupation probability $v^2(\nu h_{11/2}) = 0.0$ to 0.4 (part of fig. 2). Center: experimental energies [8] of the states of the multiplet $\pi g_{9/2}\nu h_{11/2}$ measured in $^{112}In_{63}$ (square), $^{114}In_{65}$ (triangle up), $^{116}In_{67}$ (triangle down). Right: four sets of values of residual interactions in the $\pi g_{9/2}\nu h_{11/2}$ configuration, as used in the calculation of the 3qp states of $^{113-121}In$ (see text). Empty symbols: mean experimental values of $^{112,114,116}In$, and filled symbols: extrapolated values.

energy of the 7^- level would increase and those of the 4^- and 9^- levels would decrease (see fig. 2 of ref. [5]), in agreement with the $^{114,116}In$ experimental values.

3 The $\pi g_{9/2}\nu h^2_{11/2}$ configuration in the odd-A indium isotopes

3.1 Results obtained in $^{113-121}In$

As the energies of the states of the $\pi g_{9/2}\nu h_{11/2}$ multiplet evolves as a function of the neutron number if $N > 63$, we can expect evolution in the energies of the 3qp multiplet, $\pi g_{9/2}\nu h^2_{11/2}$, in the $^{113-121}In_{64-72}$ isotopes.

The calculations described in sect. 2 have been performed for all the states of the 3qp multiplet, with spin value ranging from $9/2^+$ to $29/2^+$. Whereas only one state with spin $29/2^+$ or $27/2^+$ can be obtained from the 3qp $\pi g_{9/2}\nu h^2_{11/2}$ configuration ($J_n = 10$), we can construct two $25/2^+$ and $23/2^+$ states ($J_n = 10$ or 8), three $21/2^+$ and $19/2^+$ states ($J_n = 10$, 8 or 6), ... In these last cases, matrices of dimensions 2, 3, ... have to be diagonalized to obtain the energies of the yrast states. We only discuss below the results obtained for the states with spin $\geq 21/2$, as we cannot rely on the results obtained for the lower-spin states which are expected to have mixed configurations from other neutron sub-shells ($s_{1/2}$, $d_{3/2}$, $d_{5/2}$, and $g_{7/2}$).

As for the evolution of the $\pi g_{9/2}\nu h_{11/2}$ residual interaction as a function of the neutron number, we have used the experimental values of $^{112,114,116}In$ and some extrapolated ones in order to get the whole spectra (see the right part of fig. 3, set number 1, 2, and 3). The fourth set of values only contains extrapolated values, as the excited states of ^{118}In remain unknown at the present time. Since the calculation of the 3qp states with spin values from $21/2^+$ to $29/2^+$ does not involve the energies of the 2qp states

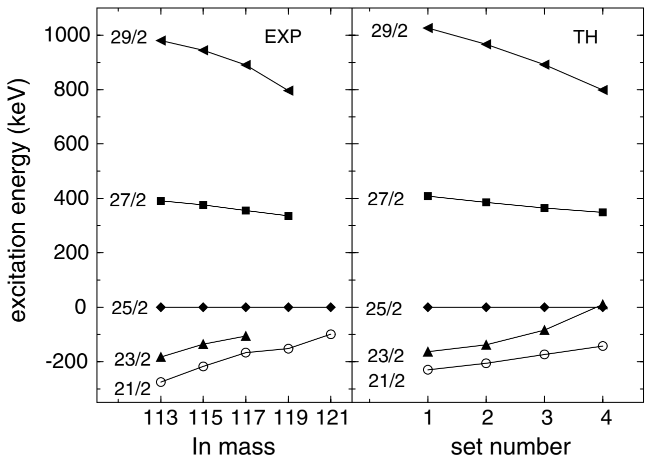

Fig. 4. Excitation energies $E(I)$ of the states of the multiplet $\pi g_{9/2}\nu h^2_{11/2}$ for $I \geq 21/2$, the energy of the $25/2^+$ state has been chosen as a reference. Left: experimental energies measured in $^{113-121}In$ [6]. Right: energies calculated using the four sets of values of residual interactions in the $\pi g_{9/2}\nu h_{11/2}$ configuration (drawn on the right side of fig. 3).

with spin value $I < 4$, it was not necessary to extrapolate the fourth set of values towards the low-spin values.

The excitation energies $E(I)$ of the states of the multiplet $\pi\nu^2$, calculated using these four sets of values of the $\pi\nu$ residual interactions, are drawn on the right side of fig. 4. The two main features of the experimental structure are well described. The narrowing of the levels comes from the decrease of the energies of the residual interaction $\pi g_{9/2}\nu h_{11/2}$. Moreover, for the fourth set of residual-interaction values the $23/2^+$ level is calculated at higher energy than the $25/2^+$ level. This inversion is mainly due to the energy gap between the 8^- and 9^- states of the 2qp configuration (on the other hand, the energy of the

7^- state does not matter). Such an inversion would explain the sudden change in the decay of the $25/2^+$ state between $A = 117$ and 119 [6]. Up to $A = 117$, it de-excites by a dipole transition towards the $23/2^+$ state. In ^{119,121}In, the observed transitions (152 keV and 99 keV) are isomeric, $T_{1/2} = 240$ and 350 ns, respectively [6], meaning a change in multipolarity, $E2$ instead of $M1$. As long as the $23/2^+$ state is located below the $25/2^+$ state, the $M1$ decay is always faster than an $E2$ decay. Therefore, the observation of the isomeric transitions implies that the order of the two states has been reversed between $A = 117$ and 119 as predicted by the calculation. Because of the inversion the $23/2^+$ state can no longer be observed experimentally.

We can conclude that the evolutions of the *relative* energies of the states of the $\pi g_{9/2}\nu h^2_{11/2}$ multiplet in the odd-A In isotopes and of the $\pi g_{9/2}\nu h_{11/2}$ multiplet in the odd-odd In isotopes are the results of the same effect, the gradual filling of the $\nu h_{11/2}$ sub-shell.

3.2 Evolution towards N = 82

In order to predict the evolution of the structure towards the neutron shell closure, the whole theoretical set of values of the residual interaction as a function of the occupation probability of the $\nu h_{11/2}$ sub-shell has been used (see fig. 2). The results are displayed in fig. 5. As said before, the states are regularly spaced, with a narrowing in energy when the occupation probability of the $\nu h_{11/2}$ sub-shell increases up to 0.5. As soon as v^2 is greater than 0.6, the $29/2^+$ level becomes the first state of the multiplet and it has to de-excite towards states with another configuration. Only the 3qp $\pi g_{9/2}\nu h_{11/2}\nu d_{3/2}$ configuration is expected at lower energy [6]. Therefore, an $E3$ transition ($29/2^+ \rightarrow 23/2^-$) can be foreseen, with the following features: i) an energy lower than 500 keV as the $\nu h_{11/2}$ and $\nu d_{3/2}$ sub-shells are close in energy (for instance, the 10^+ ($\nu h^2_{11/2}$) state is located at 488 keV above the 7^- ($\nu h_{11/2}\nu d_{3/2}$) state in ^{130}Sn$_{80}$), ii) an hindrance due to the spin flip in the $\nu h_{11/2} \rightarrow \nu d_{3/2}$ transition, $F_W \sim 100$. Therefore, a half-life about one hundred milliseconds can be expected. Such an isomeric transition has been recently measured in ^{129}In$_{80}$ [11,12] and it can be looked for in ^{127}In$_{78}$.

These calculations allow us to also discuss the case of ^{123}In$_{74}$. For $v^2 \sim 0.5$, the energies of all the states of the multiplet are very close in energy. A first isomeric state can be expected in this isotope, due to the low energy of the $E2$ transition, $25/2^+ \rightarrow 21/2^+$, as in the lighter isotopes ^{119,121}In. The $29/2^+$ level being located below the $27/2^+$ level, should be another isomeric state, because of the low energy of its $E2$ transition to the $25/2^+$ level. It is worth noting that the very low value of the pairing factor, $(u^2 - v^2) \sim 0$ in the middle of the sub- shell, has also to be considered in order to evaluate the half-lives of these two isomeric states. Indeed among the even-N and odd-N Sn isotopes which have isomeric states belonging to the $\nu h^2_{11/2}$ and $\nu h^3_{11/2}$, respectively, ^{123}Sn$_{73}$ exhibits the largest value of hindrance, $F_W = 605$, due to the mid-shell [13]. Therefore, these two positive-parity isomeric

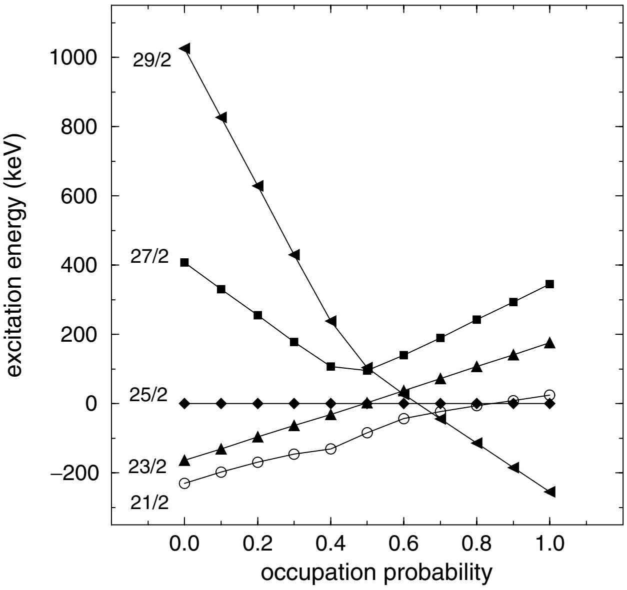

Fig. 5. Excitation energies $E(I)$ of the states of the multiplet $\pi g_{9/2}\nu h^2_{11/2}$ for $I \geq 21/2$ as functions of the occupation probability v^2 of the $\nu h_{11/2}$ sub-shell, the energy of the $25/2^+$ state being chosen as a reference.

states expected in ^{123}In should be in the millisecond scale. This is certainly the reason why this isotope could not be identified in our previous experimental work [6], the time window of the EUROBALL array being too narrow for measuring the decay of millisecond isomeric states.

4 Conclusion

In the framework of shell model calculations for three particles lying in two different orbits, the levels corresponding to the configuration $\pi g_{9/2}\nu h^2_{11/2}$ in the neutron-rich $^{113-121}$In$_{64-72}$ have been calculated. We have used two-body residual interactions which have been either extracted from experimental energies measured in neighbouring nuclei or extrapolated in order to take into account the gradual filling of the $\nu h_{11/2}$ sub-shell when the number of neutrons increases from 64 to 72. Two experimental findings, the global compression of the energies of the states with spin values between $21/2^+$ and $29/2^+$, and the disappearance of the $23/2^+$ state in ^{119,121}In, are well reproduced. Moreover, the whole calculation has allowed us to predict the existence of several long-lived isomeric states with positive parity in ^{123,125,127}In.

References

1. M. Moinester, J.P. Schiffer, W.P. Alford, Phys. Rev. **179**, 984 (1969).
2. J.P. Schiffer, Ann. Phys. (N.Y.) **66**, 798 (1971).
3. A. Molinari, M.B. Johnson, H.A. Bethe, W.M. Alberico, Nucl. Phys. A **239**, 45 (1975).

4. A. de Shalit, I. Talmi, *Nuclear Shell Theory* (Academic Press, 1963).
5. J. Van Maldeghem, K. Heyde, J. Sau, Phys. Rev. C **32**, 1067 (1985).
6. R. Lucas *et al.*, Eur. Phys. J. A **15**, 315 (2002).
7. I. Talmi, *Simple Models of Complex Nuclei* (Harwood Academic Publishers, 1993) Chapt. 31.
8. R.B. Firestone, *Table of Isotopes*, 8th edition (Wiley, New York, 1996).
9. S.P. Pandya, Phys. Rev. **103**, 956 (1956).
10. V. Paar, Nucl. Phys. A **331**, 16 (1979).
11. B. Fogelberg *et al.*, *Proceedings of the 2nd International Workshop on Nuclear Fission and Fission Product Spectroscopy, Seyssins, France (April 1998)*, AIP Conf. Proc. **447**, 191 (1998).
12. J. Genevey *et al.*, Phys. Rev. C **67**, 054312 (2003).
13. R.H. Mayer *et al.*, Phys. Lett. B **336**, 308 (1994).